An introduction to the ZX-calculus

Leo Lobski

UCL, PPLV group

28 October 2025
QInfo seminar

Outline

Process theories

The calculus

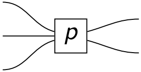
From quantum circuits to ZX

From MBQC to ZX

A logician's view

Applications

Process theories

- A *process* is drawn as a box:  ,

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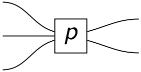
- A *process* is drawn as a box: 

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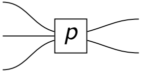
- A *process* is drawn as a box: 

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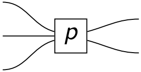
- A *process* is drawn as a box:  ,

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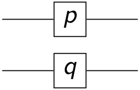
- Two processes can be *composed* (1) in parallel:  ,

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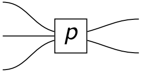
- A *process* is drawn as a box: 

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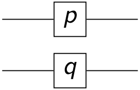
- Two processes can be *composed* (1) in parallel: 

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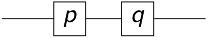
  and (2) in sequence: 

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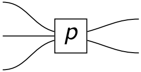
- A *process* is drawn as a box: 

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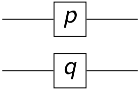
- Two processes can be *composed* (1) in parallel: 

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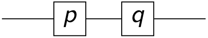
  and (2) in sequence: 

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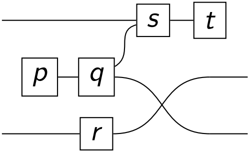

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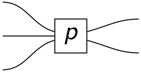
- A *process* is drawn as a box: 

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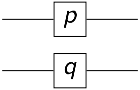
- Two processes can be *composed* (1) in parallel: 

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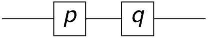
  and (2) in sequence: 

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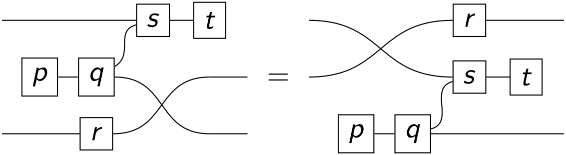

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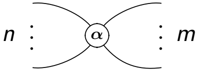
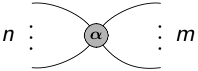


Z-spider



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Hadamard gate

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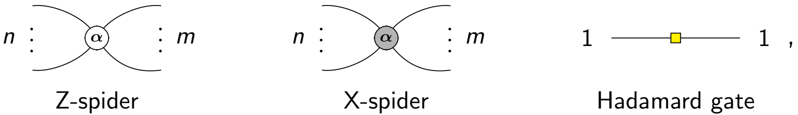

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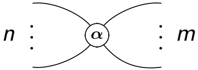

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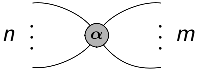

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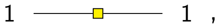

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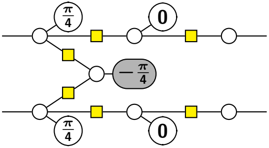

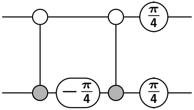

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A diagram with n inputs and m outputs is translated to a linear map $\mathbb{C}^{2^n} \to \mathbb{C}^{2^m}$,

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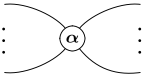

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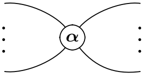

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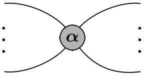

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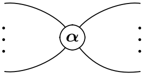

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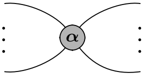

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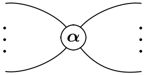

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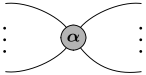

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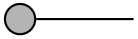
Sequential composition is interpreted as matrix multiplication, and parallel composition as the Kronecker product

Examples

 $\mapsto$

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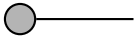 $\mapsto |0\rangle$

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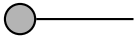
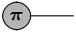
 $\mapsto |0\rangle$

 $\mapsto$

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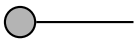
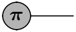
 $\mapsto |0\rangle$



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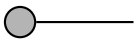
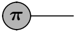
 $\mapsto |0\rangle$



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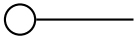 $\mapsto$

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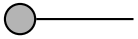
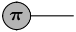
 $\mapsto |0\rangle$



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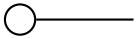 $\mapsto |+\rangle$

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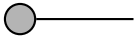
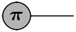
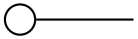
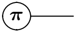
 $\mapsto |0\rangle$

 $\mapsto |1\rangle$

 $\mapsto |+\rangle$

 $\mapsto$

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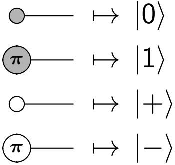
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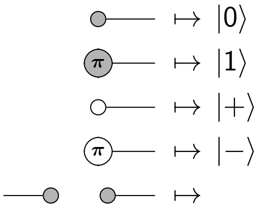

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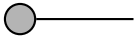
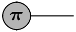
 $\mapsto |0\rangle$



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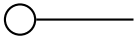
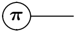
 $\mapsto |+\rangle$



$\mapsto |-\rangle$

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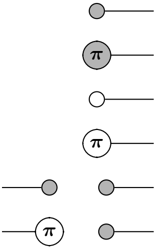

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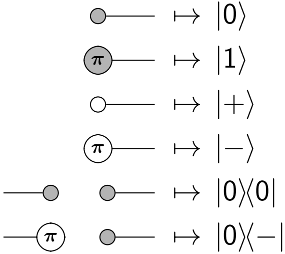
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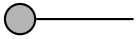
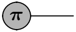
 $\mapsto |0\rangle$



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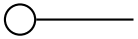
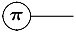
 $\mapsto |+\rangle$



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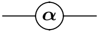

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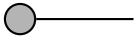
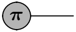
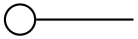
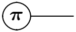
 $\mapsto |0\rangle$

 $\mapsto |1\rangle$

 $\mapsto |+\rangle$

 $\mapsto |-\rangle$

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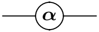 $\mapsto |0\rangle\langle 0| + e^{i\alpha} |1\rangle\langle 1|$

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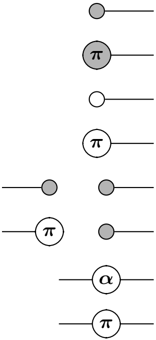

$\mapsto |0\rangle$

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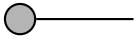
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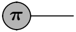

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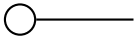
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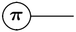

$\mapsto |-\rangle$


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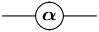

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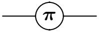

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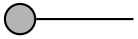
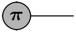
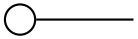
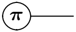
 $\mapsto |0\rangle$

 $\mapsto |1\rangle$

 $\mapsto |+\rangle$

 $\mapsto |-\rangle$

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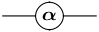
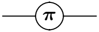
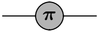
 $\mapsto |0\rangle\langle 0| + e^{i\alpha} |1\rangle\langle 1|$

 $\mapsto |0\rangle\langle 0| - |1\rangle\langle 1|$

 $\mapsto$

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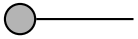
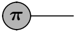
 $\mapsto |0\rangle$



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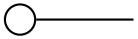
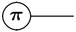
 $\mapsto |+\rangle$



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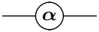

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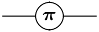

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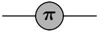

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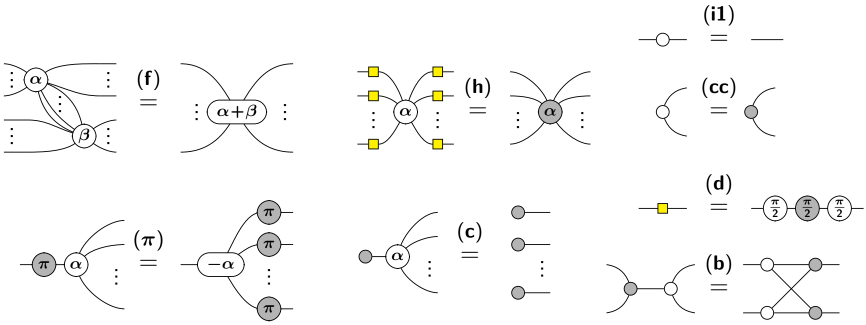

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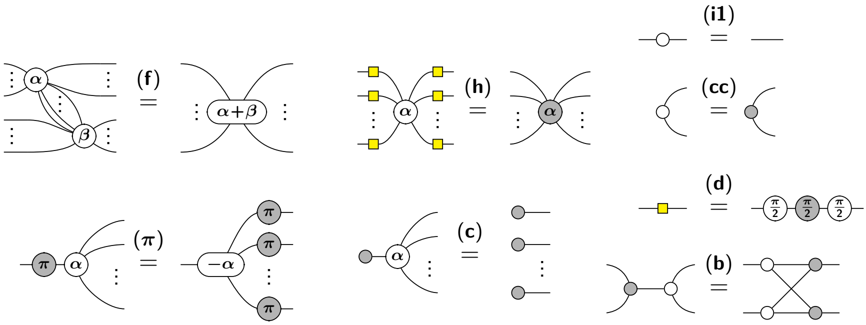

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The equations identify linear maps *up to a global non-zero scalar*!!!

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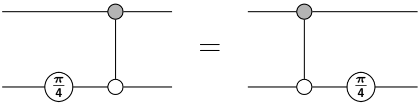

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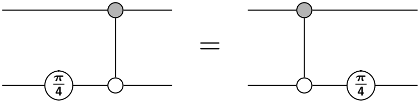

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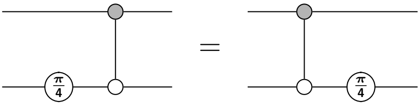

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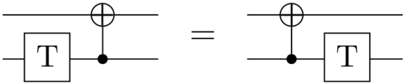
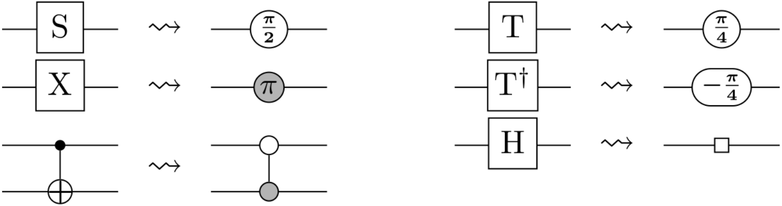


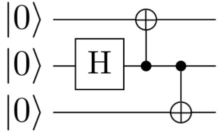
From quantum circuits to ZX



[1]

[1] Image credit: John van de Wetering

Example: GHZ state



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- X_n^s or Z_n^s – apply Pauli-X or Pauli-Z operator if $s = 1$ (do nothing if $s = 0$)

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Example

Consider the following pattern:

$$N_2 E_{12} M_1^{XY,0} X_2^{s_1}$$

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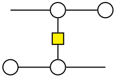
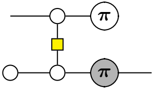


$s_1 = 0$



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- *Sound*: If two diagrams are equal, then the corresponding linear maps are equal,
- *Universal*: Every linear map $\mathbb{C}^{2^n} \to \mathbb{C}^{2^m}$ can be represented as a diagram,
- *Complete*: If two linear maps are equal, then the corresponding diagrams are equal[3].

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Applications

- Classical simulation of quantum circuits

- Circuit optimisation

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  - Staudacher, Schmid, Zeiher, Wille and Kranzlmüller: *Multi-controlled phase gate synthesis with ZX-calculus applied to neutral atom hardware*, EPTCS, 2024

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  - Cowtan, Dilkes, Duncan, Simmons and Sivarajah: *Phase Gadget Synthesis for Shallow Circuits*, EPTCS 2020

Further reading

- ZX-website: *zxcalculus.com*
- John van de Wetering: *ZX-calculus for the working quantum computer scientist*, arXiv:2012.13966
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