There and back again: A circuit extraction tale

Miriam Backens, Hector Miller-Bakewell, Giovanni de Felice,
Leo Lobski and John van de Wetering

QPL 2020

Two paradigms of quantum computation

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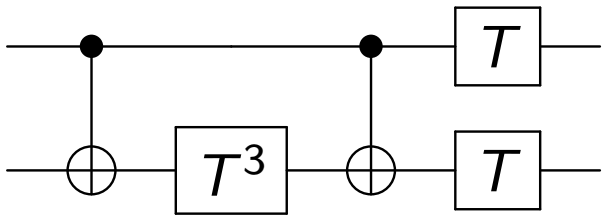
Gate based



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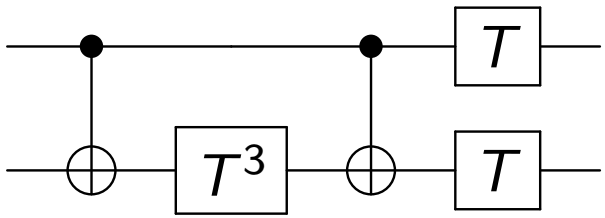

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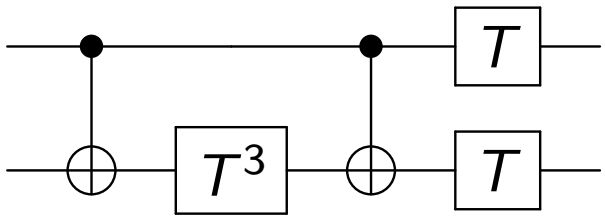

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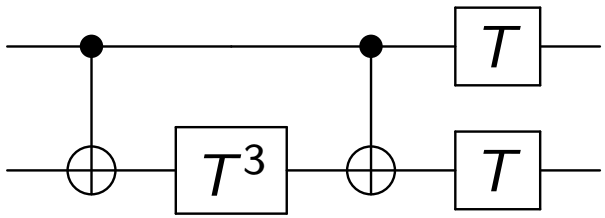

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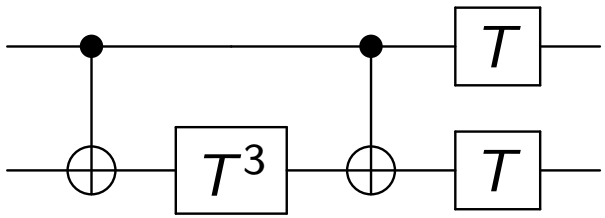

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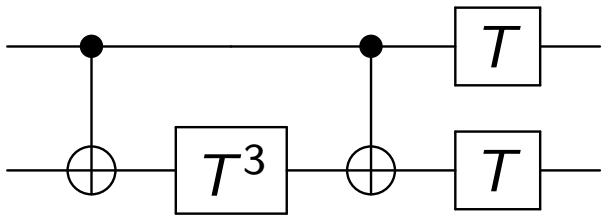

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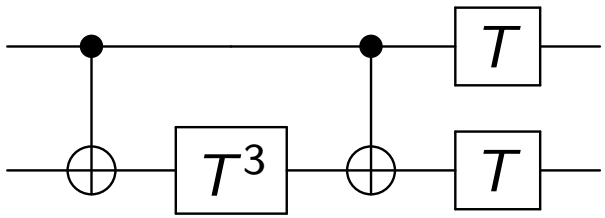

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- Computation performed by measurements
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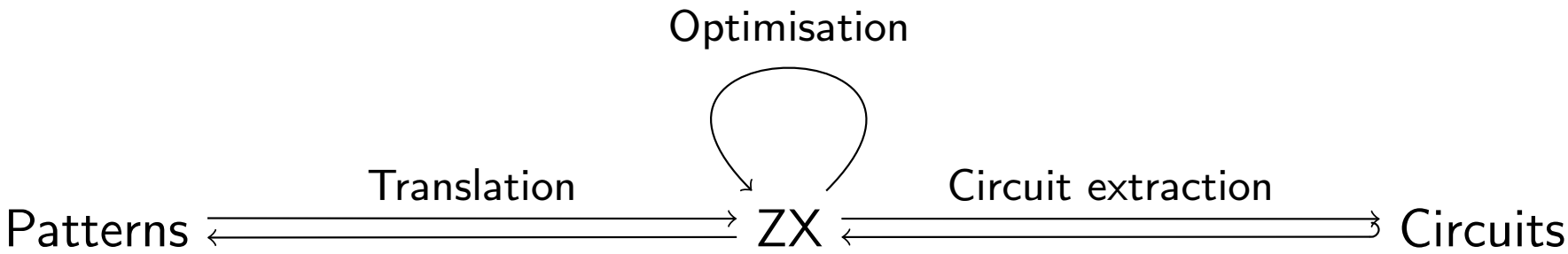

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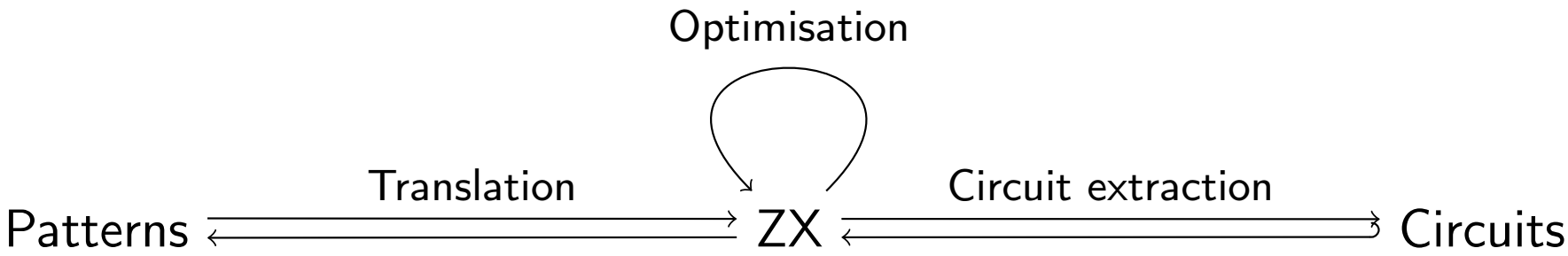

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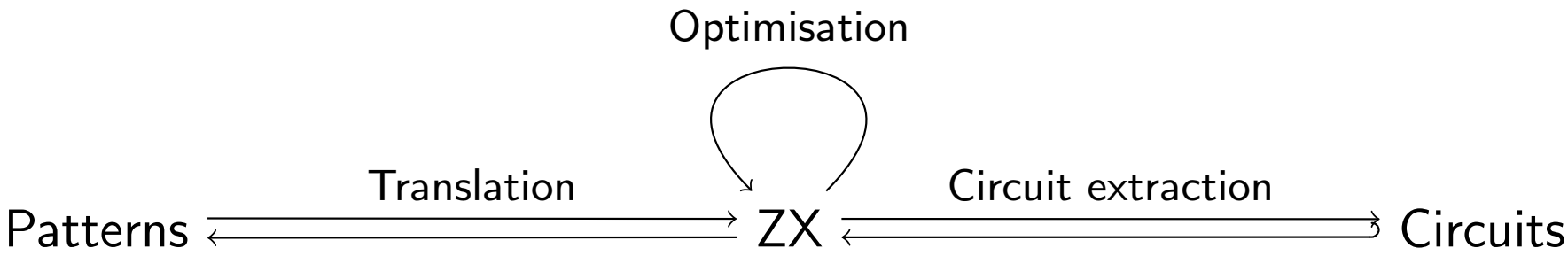

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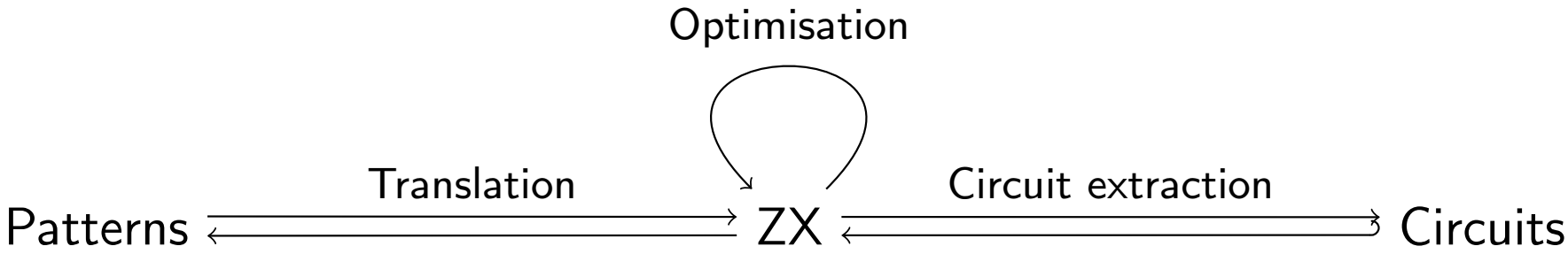

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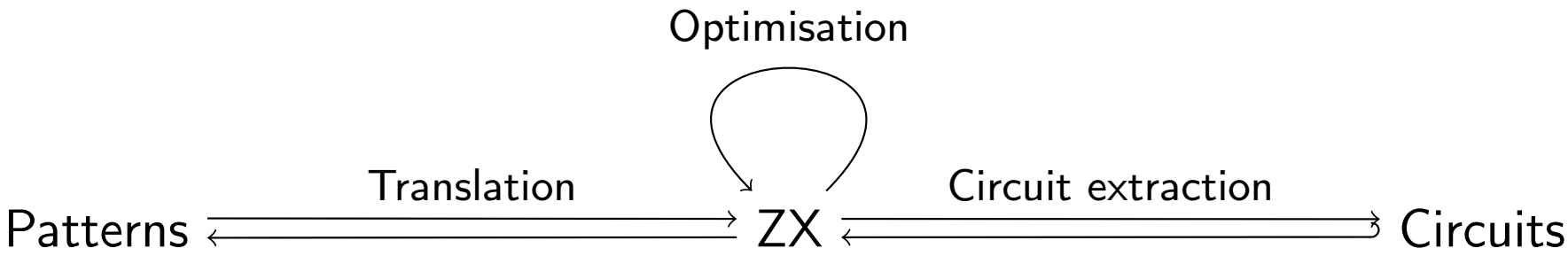

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- ZX-calculus as a tool for translation
- Circuit extraction algorithm for measurement patterns whose translation has a *gflow*
  - This generalises the algorithm by Duncan, Kissinger, Perdrix and van de Wetering (2019)

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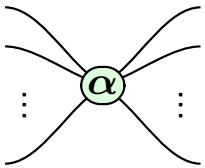

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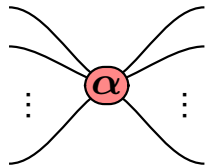

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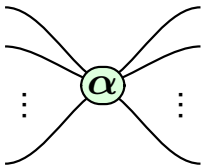

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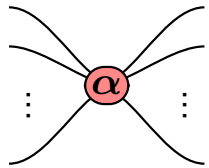

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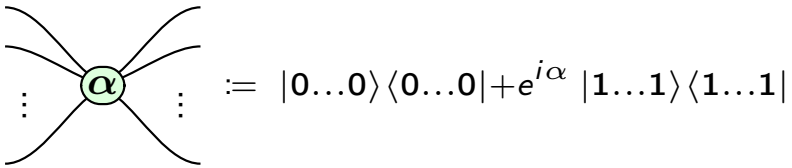

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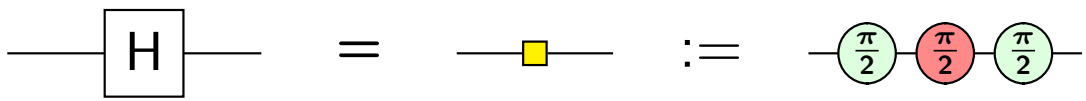

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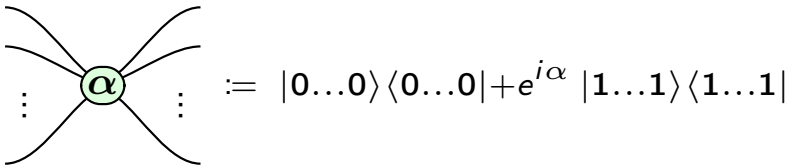

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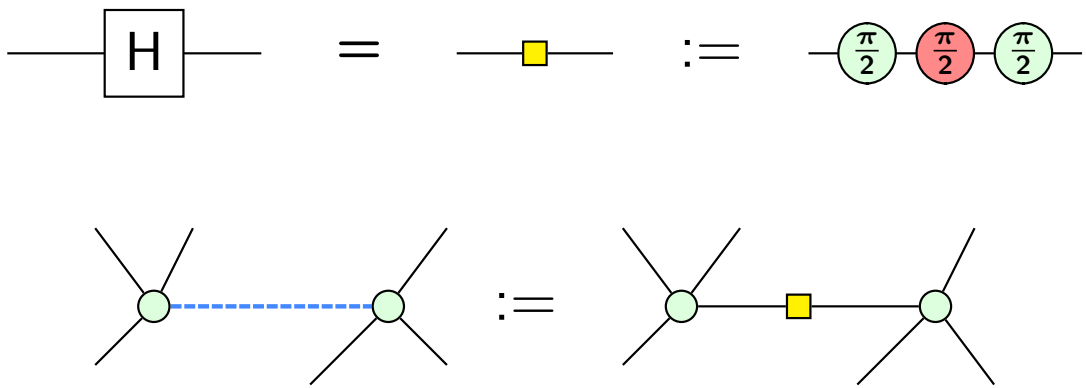
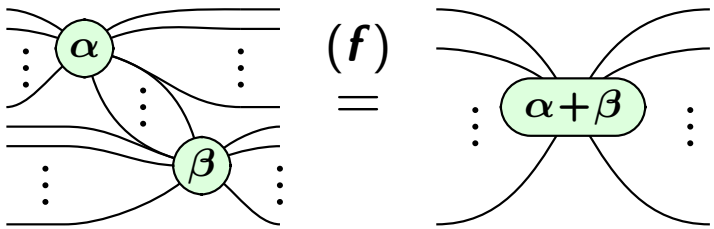
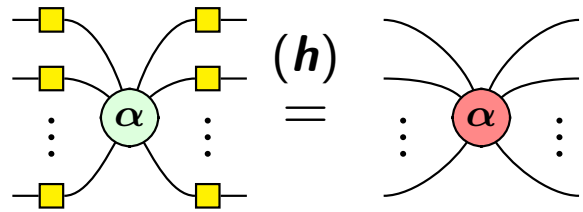
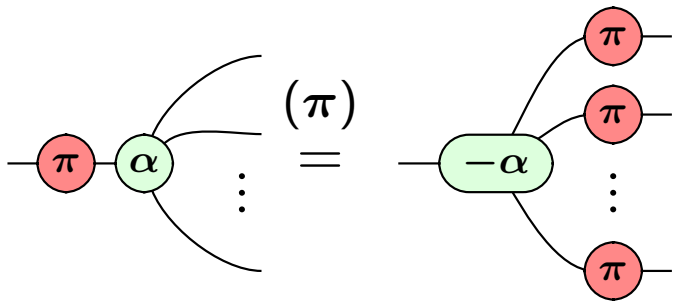
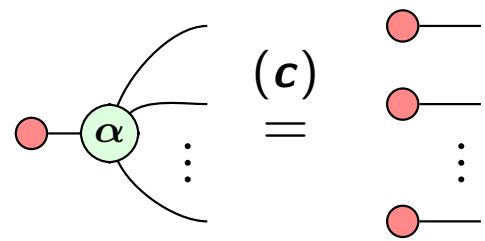


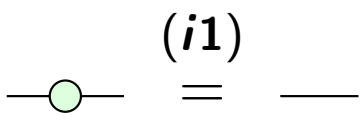
Some ZX rules

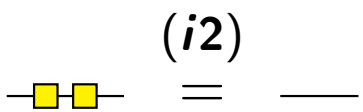


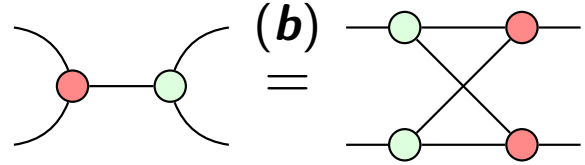












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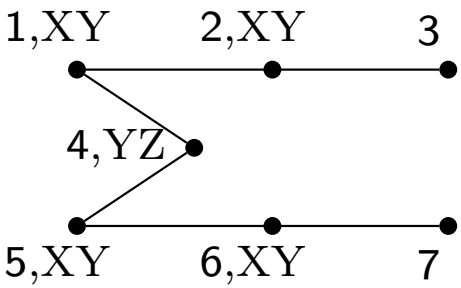

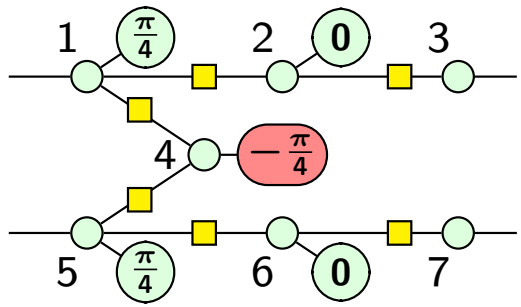

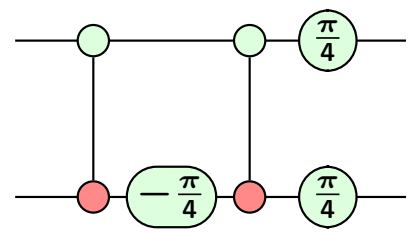

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- Idea: order vertices in such a way that any measurement error can either be corrected in the future, or cancels out with another measurement error

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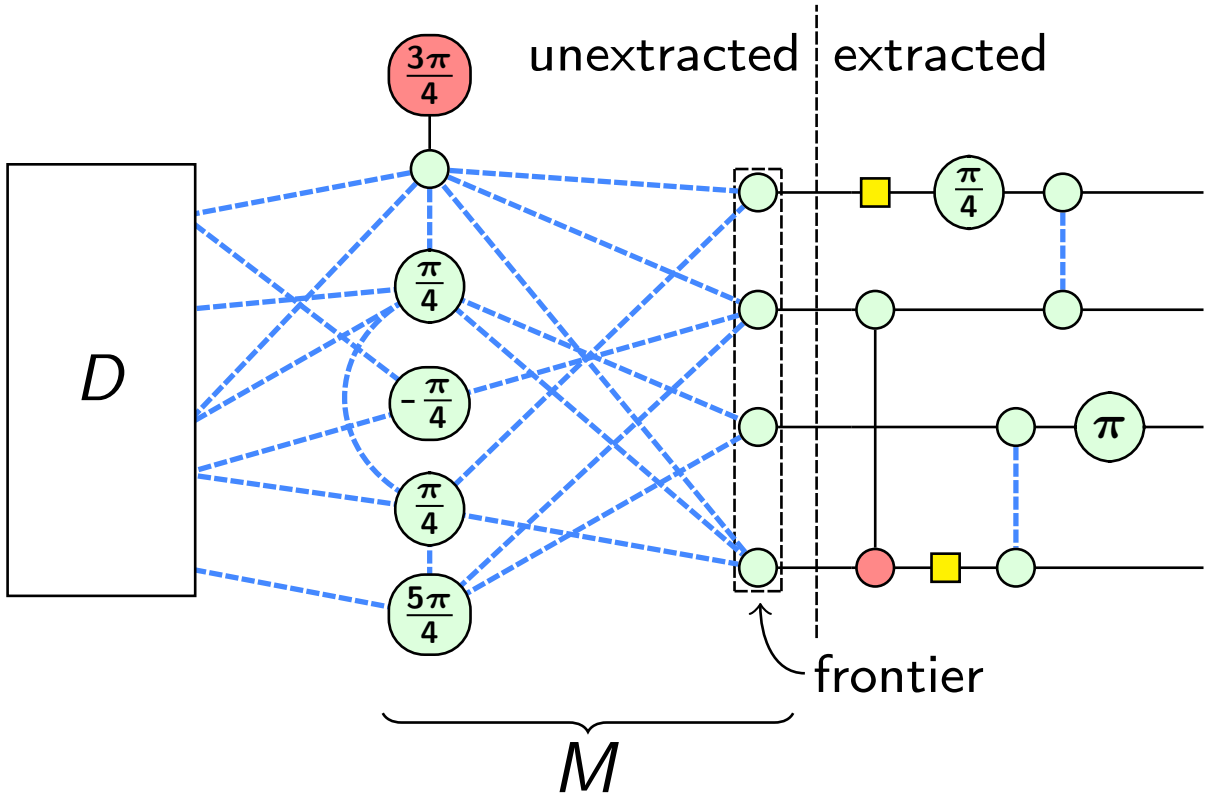

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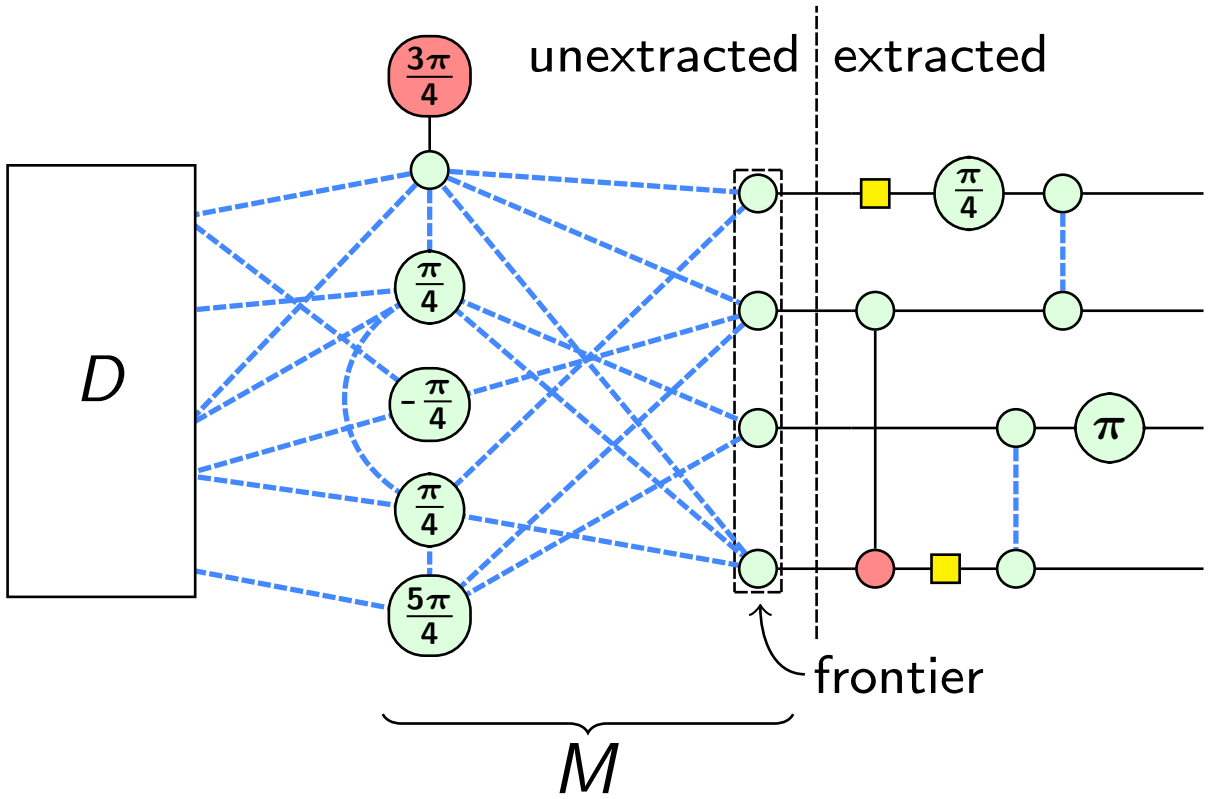

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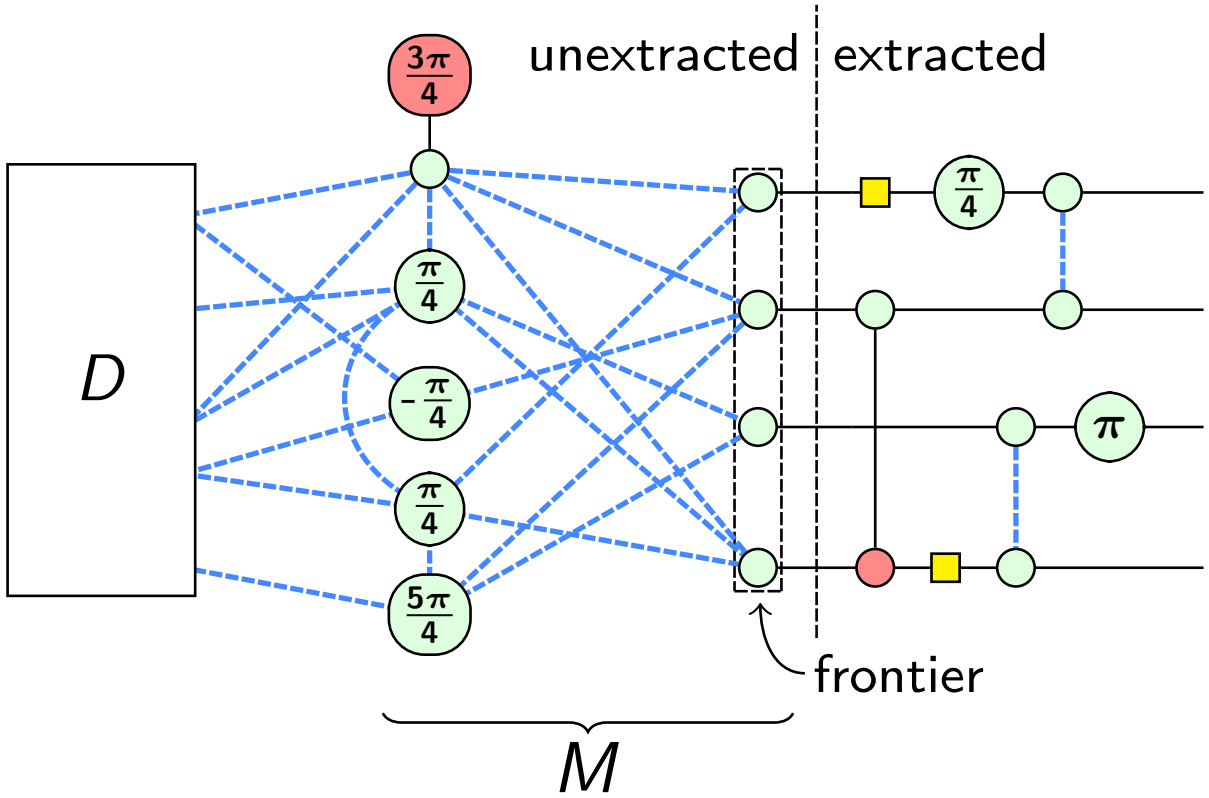

- Apply local rewrites to the unextracted part
- When the frontier vertices look like a circuit, move them to the extracted part

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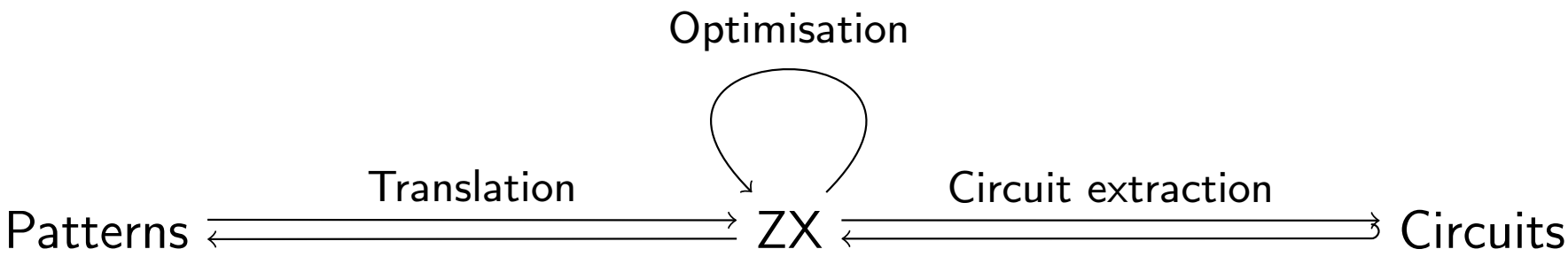

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- It may thus be useful to replace this part of the algorithm with one incorporating routing, as developed e.g. by Kissinger and Meijer-van de Griend (2019)

References

- Miriam Backens, Hector Miller-Bakewell, Giovanni de Felice, Leo Lobski and John van de Wetering. *There and back again: A circuit rewriting tale*. arXiv:2003.01664, 2020.
- Ross Duncan, Alex Kissinger, Simon Perdrix and John van de Wetering. *Graph-theoretic simplification of quantum circuits with the ZX-calculus*. arxiv:1902.03178, 2019.
- Aleks Kissinger and Arianne Meijer-van de Griend. *CNOT circuit extraction for topologically-constrained quantum memories*. arXiv:1904.00633, 2019.
- Daniel E. Browne, Elham Kashefi, Mehdi Mhalla and Simon Perdrix. *Generalized flow and determinism in measurement-based quantum computation*. New Journal of Physics, 9(8):250, 2007.
- V. Danos and E. Kashefi. *Determinism in the one-way model*. Phys. Rev. A, 74 (052310), 2006.

Thank you for your attention!