Towards a Combinatorial Representation of First-Order Bicategories

Leo Lobski, Ralph Sarkis, Paul Wilson and Fabio Zanasi

DIALOCO 2026
Lisbon, 19 July 2026

Outline

1. Background: Hypergraphs & regular logic
2. Branching hypergraphs & first-order logic

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$.

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$. We fix a relational signature Σ for the entirety of the talk.

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$. We fix a relational signature Σ for the entirety of the talk.

Definition (Hyp)

A Σ*-labelled hypergraph with equality* (V, U, E, H, s, t, L) is:

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$. We fix a relational signature Σ for the entirety of the talk.

Definition (Hyp)

A Σ*-labelled hypergraph with equality* (V, U, E, H, s, t, L) is:

- a set of *vertices* V,

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$. We fix a relational signature Σ for the entirety of the talk.

Definition (Hyp)

A Σ*-labelled hypergraph with equality* (V, U, E, H, s, t, L) is:

- a set of *vertices* V,
- a subset $U \subseteq V$,

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$. We fix a relational signature Σ for the entirety of the talk.

Definition (Hyp)

A Σ-*labelled hypergraph with equality* (V, U, E, H, s, t, L) is:

- a set of *vertices* V,
- a subset $U \subseteq V$,
- a reflexive and symmetric binary relation $E \subseteq V \times V$ called *equality edges*,

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$. We fix a relational signature Σ for the entirety of the talk.

Definition (Hyp)

A Σ-*labelled hypergraph with equality* (V, U, E, H, s, t, L) is:

- a set of *vertices* V,
- a subset $U \subseteq V$,
- a reflexive and symmetric binary relation $E \subseteq V \times V$ called *equality edges*,
- a set of *hyperedges* H,

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$. We fix a relational signature Σ for the entirety of the talk.

Definition (Hyp)

A Σ*-labelled hypergraph with equality* (V, U, E, H, s, t, L) is:

- a set of *vertices* V,
- a subset $U \subseteq V$,
- a reflexive and symmetric binary relation $E \subseteq V \times V$ called *equality edges*,
- a set of *hyperedges* H,
- *source* and *target* functions $s, t : H \to V^*$,

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$. We fix a relational signature Σ for the entirety of the talk.

Definition (Hyp)

A Σ-*labelled hypergraph with equality* (V, U, E, H, s, t, L) is:

- a set of *vertices* V,
- a subset $U \subseteq V$,
- a reflexive and symmetric binary relation $E \subseteq V \times V$ called *equality edges*,
- a set of *hyperedges* H,
- *source* and *target* functions $s, t : H \to V^*$,
- a *labelling* function $L : H \to \Sigma$ such that $\mathtt{ar}(L(h)) = |s(h)|$ and $\mathtt{coar}(L(h)) = |t(h)|$.

Hypergraphs over a relational signature

A (directed) *relational signature* $(\Sigma, \mathtt{ar}, \mathtt{coar})$ consists of a set Σ and functions $\mathtt{ar}, \mathtt{coar} : \Sigma \to \mathbb{N}$. We fix a relational signature Σ for the entirety of the talk.

Definition (Hyp)

A Σ-*labelled hypergraph with equality* (V, U, E, H, s, t, L) is:

- a set of *vertices* V,
- a subset $U \subseteq V$,
- a reflexive and symmetric binary relation $E \subseteq V \times V$ called *equality edges*,
- a set of *hyperedges* H,
- *source* and *target* functions $s, t : H \to V^*$,
- a *labelling* function $L : H \to \Sigma$ such that $\mathtt{ar}(L(h)) = |s(h)|$ and $\mathtt{coar}(L(h)) = |t(h)|$.

A hyp is *discrete* if E is the diagonal relation.

From hypergraphs to first-order logic

Assign a FOL formula to hyp $\mathcal{H} = (V, U, E, H, s, t, L)$:

From hypergraphs to first-order logic

Assign a FOL formula to hyp $\mathcal{H} = (V, U, E, H, s, t, L)$:

$$\varphi_E \coloneqq \bigwedge_{(u,v)\in E} u = v, \qquad \varphi_H \coloneqq \bigwedge_{h\in H} L(h)\left(s(h), t(h)\right),$$

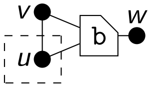
$$\varphi_{\mathcal{H}} \coloneqq \exists\{v : v \in V \setminus U\}.\varphi_E \wedge \varphi_H.$$

From hypergraphs to first-order logic

Assign a FOL formula to hyp $\mathcal{H} = (V, U, E, H, s, t, L)$:

$$\varphi_E \coloneqq \bigwedge_{(u,v)\in E} u = v, \qquad \varphi_H \coloneqq \bigwedge_{h\in H} L(h)\left(s(h), t(h)\right),$$

$$\varphi_{\mathcal{H}} \coloneqq \exists\{v : v \in V \setminus U\}.\varphi_E \wedge \varphi_H.$$

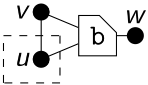
(1) 

From hypergraphs to first-order logic

Assign a FOL formula to hyp $\mathcal{H} = (V, U, E, H, s, t, L)$:

$$\varphi_E \coloneqq \bigwedge_{(u,v)\in E} u = v, \qquad \varphi_H \coloneqq \bigwedge_{h\in H} L(h)\,(s(h), t(h))\,,$$

$$\varphi_{\mathcal{H}} \coloneqq \exists\{v : v \in V \setminus U\}.\varphi_E \wedge \varphi_H.$$

(1) 

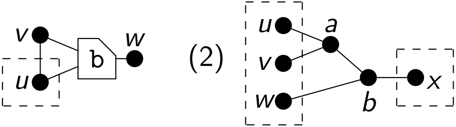
(1) $\exists v, w.(v = u) \wedge \mathtt{b}(vu, w)$

From hypergraphs to first-order logic

Assign a FOL formula to hyp $\mathcal{H} = (V, U, E, H, s, t, L)$:

$$\varphi_E \coloneqq \bigwedge_{(u,v)\in E} u = v, \qquad \varphi_H \coloneqq \bigwedge_{h\in H} L(h)\,(s(h), t(h))\,,$$

$$\varphi_{\mathcal{H}} \coloneqq \exists\{v : v \in V \setminus U\}.\varphi_E \wedge \varphi_H.$$

(1) 

(2)

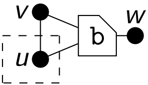
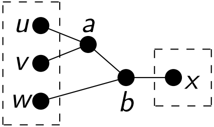
(1) $\exists v, w.(v = u) \wedge \mathtt{b}(vu, w)$

From hypergraphs to first-order logic

Assign a FOL formula to hyp $\mathcal{H} = (V, U, E, H, s, t, L)$:

$$\varphi_E := \bigwedge_{(u,v)\in E} u = v, \qquad \varphi_H := \bigwedge_{h\in H} L(h)\,(s(h), t(h))\,,$$

$$\varphi_{\mathcal{H}} := \exists\{v : v \in V \setminus U\}.\varphi_E \wedge \varphi_H.$$

(1)  (2) 

(1) $\exists v, w.(v = u) \wedge \mathtt{b}(vu, w)$

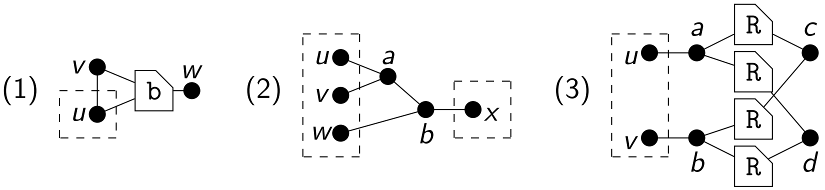
(2) $\exists a, b.(a = u) \wedge (a = v) \wedge (a = b) \wedge (b = w) \wedge (b = x)$

From hypergraphs to first-order logic

Assign a FOL formula to hyp $\mathcal{H} = (V, U, E, H, s, t, L)$:

$$\varphi_E \coloneqq \bigwedge_{(u,v)\in E} u = v, \qquad \varphi_H \coloneqq \bigwedge_{h\in H} L(h)\,(s(h), t(h))\,,$$

$$\varphi_{\mathcal{H}} \coloneqq \exists\{v : v \in V \setminus U\}.\varphi_E \wedge \varphi_H.$$



(1) $\exists v, w.(v = u) \wedge \mathtt{b}(vu, w)$

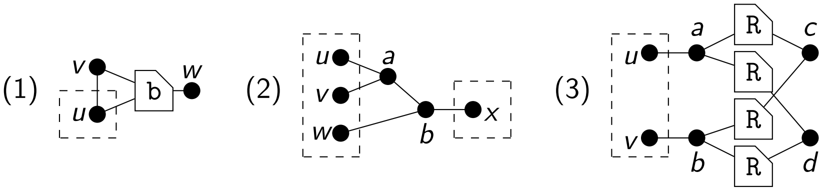
(2) $\exists a, b.(a = u) \wedge (a = v) \wedge (a = b) \wedge (b = w) \wedge (b = x)$

From hypergraphs to first-order logic

Assign a FOL formula to hyp $\mathcal{H} = (V, U, E, H, s, t, L)$:

$$\varphi_E \coloneqq \bigwedge_{(u,v)\in E} u = v, \qquad\qquad \varphi_H \coloneqq \bigwedge_{h\in H} L(h)\,(s(h), t(h)),$$

$$\varphi_{\mathcal{H}} \coloneqq \exists\{v : v \in V \setminus U\}.\varphi_E \wedge \varphi_H.$$



(1) $\exists v, w.(v = u) \wedge \mathtt{b}(vu, w)$

(2) $\exists a, b.(a = u) \wedge (a = v) \wedge (a = b) \wedge (b = w) \wedge (b = x)$

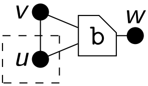
(3) $\exists a, b, c, d.(a = u) \wedge (b = v) \wedge \mathtt{R}(a, c) \wedge \mathtt{R}(a, d) \wedge \mathtt{R}(b, c) \wedge \mathtt{R}(b, d)$

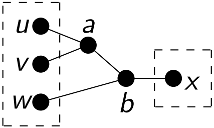
From hypergraphs to first-order logic

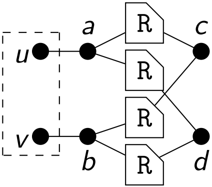
Assign a FOL formula to hyp $\mathcal{H} = (V, U, E, H, s, t, L)$:

$$\varphi_E \coloneqq \bigwedge_{(u,v)\in E} u = v, \qquad \varphi_H \coloneqq \bigwedge_{h\in H} L(h)\left(s(h), t(h)\right),$$

$$\varphi_{\mathcal{H}} \coloneqq \exists\{v : v \in V \setminus U\}.\varphi_E \wedge \varphi_H.$$

(1) 

(2) 

(3) 

(1) $\exists v, w.(v = u) \wedge \mathtt{b}(vu, w)$

(2) $\exists a, b.(a = u) \wedge (a = v) \wedge (a = b) \wedge (b = w) \wedge (b = x)$

(3) $\exists a, b, c, d.(a = u) \wedge (b = v) \wedge \mathtt{R}(a, c) \wedge \mathtt{R}(a, d) \wedge \mathtt{R}(b, c) \wedge \mathtt{R}(b, d)$

Note: $\mathrm{FV}(\varphi_{\mathcal{H}}) = U$.

Satisfaction

- An *assignment* from a hyp $\mathcal{H}$ to a discrete hyp $\mathcal{H}'$ is a function $\theta : U \to V'$.

Satisfaction

- An *assignment* from a hyp $\mathcal{H}$ to a discrete hyp $\mathcal{H}'$ is a function $\theta : U \to V'$.
- The assignment *satisfies* the hyp ($\theta \vDash \mathcal{H}$) if there is a morphism of hyps $f : \mathcal{H} \to \mathcal{H}'$ such that $f|_U = \theta$.

Satisfaction

- An *assignment* from a hyp $\mathcal{H}$ to a discrete hyp $\mathcal{H}'$ is a function $\theta : U \to V'$.
- The assignment *satisfies* the hyp ($\theta \vDash \mathcal{H}$) if there is a morphism of hyps $f : \mathcal{H} \to \mathcal{H}'$ such that $f|_U = \theta$.

Proposition

$\theta \vDash \mathcal{H}$ *if an only if* $\mathcal{H}'$ *is a model of* $\varphi_{\mathcal{H}}$.

Models of regular logic

More is true (Bonchi, Seeber and Sobociński, 2018):

Models of regular logic

More is true (Bonchi, Seeber and Sobociński, 2018):

- cospans of hypergraphs give the free cartesian bicategory over Σ,

Models of regular logic

More is true (Bonchi, Seeber and Sobociński, 2018):

- cospans of hypergraphs give the free cartesian bicategory over Σ,
- morphisms between the hypergraphs (2-cells) correspond to entailment between formulas: there is a morphism $\mathcal{H} \to \mathcal{H}'$ if and only if $\varphi_{\mathcal{H}'} \Rightarrow \varphi_{\mathcal{H}}$,

Models of regular logic

More is true (Bonchi, Seeber and Sobociński, 2018):

- cospans of hypergraphs give the free cartesian bicategory over Σ,
- morphisms between the hypergraphs (2-cells) correspond to entailment between formulas: there is a morphism $\mathcal{H} \to \mathcal{H}'$ if and only if $\varphi_{\mathcal{H}'} \Rightarrow \varphi_{\mathcal{H}}$,
- completeness with respect to relational composition, and hence with respect to the *regular* fragment of FOL ($\exists$, $\wedge$, $\top$), follows.

Models of regular logic

More is true (Bonchi, Seeber and Sobociński, 2018):

- cospans of hypergraphs give the free cartesian bicategory over Σ,
- morphisms between the hypergraphs (2-cells) correspond to entailment between formulas: there is a morphism $\mathcal{H} \to \mathcal{H}'$ if and only if $\varphi_{\mathcal{H}'} \Rightarrow \varphi_{\mathcal{H}}$,
- completeness with respect to relational composition, and hence with respect to the *regular* fragment of FOL ($\exists$, $\wedge$, $\top$), follows.

This work: extend this to full first-order logic.

Branching hypergraphs

- The set of *branches* is given by $\{-1\} \cup \mathbb{N}^*$;

Branching hypergraphs

- The set of *branches* is given by $\{-1\} \cup \mathbb{N}^*$; it is partially ordered by $-1 \leq \gamma$ and by extension.

Branching hypergraphs

- The set of *branches* is given by $\{-1\} \cup \mathbb{N}^*$; it is partially ordered by $-1 \leq \gamma$ and by extension.
- A *tree* is a finite set of branches that contains ε and is downwards closed.

Branching hypergraphs

- The set of *branches* is given by $\{-1\} \cup \mathbb{N}^*$; it is partially ordered by $-1 \leq \gamma$ and by extension.
- A *tree* is a finite set of branches that contains ε and is downwards closed.
- A *branching hypergraph* (*byp*) is a tree T and for each branch $\gamma \in T$ a hyp $\mathcal{B}(\gamma) = (V(\gamma), U(\gamma), E(\gamma), H(\gamma), s(\gamma), t(\gamma), L(\gamma))$ such that

Branching hypergraphs

- The set of *branches* is given by $\{-1\} \cup \mathbb{N}^*$; it is partially ordered by $-1 \leq \gamma$ and by extension.
- A *tree* is a finite set of branches that contains ε and is downwards closed.
- A *branching hypergraph* (*byp*) is a tree T and for each branch $\gamma \in T$ a hyp $\mathcal{B}(\gamma) = (V(\gamma), U(\gamma), E(\gamma), H(\gamma), s(\gamma), t(\gamma), L(\gamma))$ such that
  - $U(-1) = V(-1)$,

Branching hypergraphs

- The set of *branches* is given by $\{-1\} \cup \mathbb{N}^*$; it is partially ordered by $-1 \leq \gamma$ and by extension.
- A *tree* is a finite set of branches that contains ε and is downwards closed.
- A *branching hypergraph* (*byp*) is a tree T and for each branch $\gamma \in T$ a hyp $\mathcal{B}(\gamma) = (V(\gamma), U(\gamma), E(\gamma), H(\gamma), s(\gamma), t(\gamma), L(\gamma))$ such that
  - $U(-1) = V(-1)$,
  - $E(-1) = H(-1) = \varnothing$,

Branching hypergraphs

- The set of *branches* is given by $\{-1\} \cup \mathbb{N}^*$; it is partially ordered by $-1 \leq \gamma$ and by extension.
- A *tree* is a finite set of branches that contains ε and is downwards closed.
- A *branching hypergraph* (*byp*) is a tree T and for each branch $\gamma \in T$ a hyp $\mathcal{B}(\gamma) = (V(\gamma), U(\gamma), E(\gamma), H(\gamma), s(\gamma), t(\gamma), L(\gamma))$ such that
  - $U(-1) = V(-1)$,
  - $E(-1) = H(-1) = \varnothing$,
  - if $\gamma \leq \delta$, then $V(\gamma) \subseteq V(\delta)$,

Branching hypergraphs

- The set of *branches* is given by $\{-1\} \cup \mathbb{N}^*$; it is partially ordered by $-1 \leq \gamma$ and by extension.
- A *tree* is a finite set of branches that contains ε and is downwards closed.
- A *branching hypergraph* (*byp*) is a tree T and for each branch $\gamma \in T$ a hyp $\mathcal{B}(\gamma) = (V(\gamma), U(\gamma), E(\gamma), H(\gamma), s(\gamma), t(\gamma), L(\gamma))$ such that
  - $U(-1) = V(-1)$,
  - $E(-1) = H(-1) = \varnothing$,
  - if $\gamma \leq \delta$, then $V(\gamma) \subseteq V(\delta)$,
  - $U(\gamma n) = V(\gamma)$ and $U(\varepsilon) = V(-1)$.

Branching hypergraphs

- The set of *branches* is given by $\{-1\} \cup \mathbb{N}^*$; it is partially ordered by $-1 \leq \gamma$ and by extension.
- A *tree* is a finite set of branches that contains ε and is downwards closed.
- A *branching hypergraph* (*byp*) is a tree T and for each branch $\gamma \in T$ a hyp
$\mathcal{B}(\gamma) = (V(\gamma), U(\gamma), E(\gamma), H(\gamma), s(\gamma), t(\gamma), L(\gamma))$ such that
  - $U(-1) = V(-1)$,
  - $E(-1) = H(-1) = \varnothing$,
  - if $\gamma \leq \delta$, then $V(\gamma) \subseteq V(\delta)$,
  - $U(\gamma n) = V(\gamma)$ and $U(\varepsilon) = V(-1)$.
- For $\gamma \in T \cap \mathbb{N}^*$, the *restriction* byp $\mathcal{B}|_\gamma$ is defined by making the parent of γ the root (-1) and γ the empty branch (ε).

From branching hypergraphs to first-order logic

Each byp $\mathcal{B}$ induces the FOL formula:

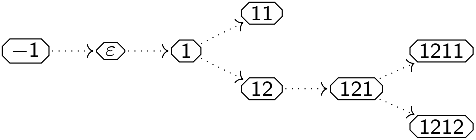
$$\varphi_{\mathcal{B}} \coloneqq \exists \{v : v \in V(\varepsilon) \setminus U(\varepsilon)\} . \varphi_{E(\varepsilon)} \wedge \varphi_{H(\varepsilon)} \wedge \bigwedge_{n \in T \cap \mathbb{N}} \neg \varphi_{\mathcal{B}|_n}.$$

From branching hypergraphs to first-order logic

Each byp $\mathcal{B}$ induces the FOL formula:

$$\varphi_{\mathcal{B}} \coloneqq \exists \{v : v \in V(\varepsilon) \setminus U(\varepsilon)\} . \varphi_{E(\varepsilon)} \wedge \varphi_{H(\varepsilon)} \wedge \bigwedge_{n \in T \cap \mathbb{N}} \neg \varphi_{\mathcal{B}|_n}.$$

Example tree:

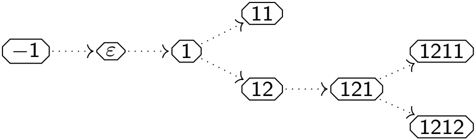


From branching hypergraphs to first-order logic

Each byp $\mathcal{B}$ induces the FOL formula:

$$\varphi_{\mathcal{B}} \coloneqq \exists \{v : v \in V(\varepsilon) \setminus U(\varepsilon)\} . \varphi_{E(\varepsilon)} \wedge \varphi_{H(\varepsilon)} \wedge \bigwedge_{n \in T \cap \mathbb{N}} \neg \varphi_{\mathcal{B}|_n}.$$

Example tree:

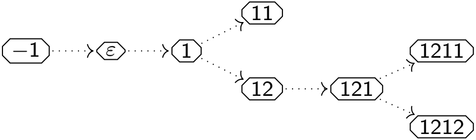


Consider the byp (represented as a *flattened* byp, or *fyp*):

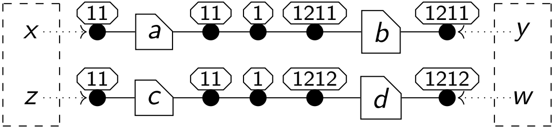
From branching hypergraphs to first-order logic

Each byp $\mathcal{B}$ induces the FOL formula:

$$\varphi_{\mathcal{B}} \coloneqq \exists \{v : v \in V(\varepsilon) \setminus U(\varepsilon)\} . \varphi_{E(\varepsilon)} \wedge \varphi_{H(\varepsilon)} \wedge \bigwedge_{n \in T \cap \mathbb{N}} \neg \varphi_{\mathcal{B}|_n}.$$

Example tree: 

Consider the byp (represented as a *flattened* byp, or *fyp*):

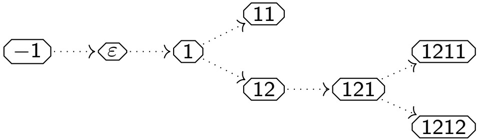


From branching hypergraphs to first-order logic

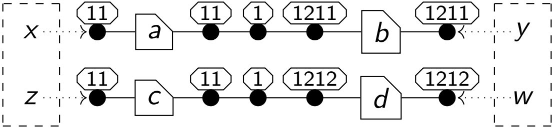
Each byp $\mathcal{B}$ induces the FOL formula:

$$\varphi_{\mathcal{B}} := \exists \{ v : v \in V(\varepsilon) \setminus U(\varepsilon) \} . \varphi_{E(\varepsilon)} \wedge \varphi_{H(\varepsilon)} \wedge \bigwedge_{n \in T \cap \mathbb{N}} \neg \varphi_{\mathcal{B}|_n}.$$

Example tree:



Consider the byp (represented as a *flattened* byp, or *fyp*):



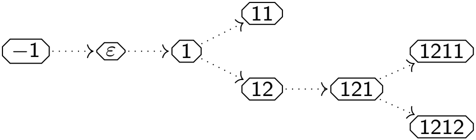
The FOL formula obtained is:

$$\neg \exists i, j. \neg \left(a(x, i) \wedge c(z, j) \right) \wedge \neg\neg \left(\neg b(i, y) \wedge \neg d(j, w) \right),$$

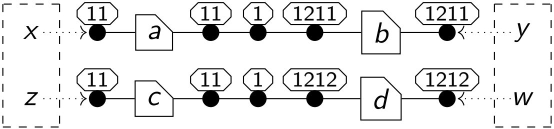
From branching hypergraphs to first-order logic

Each byp $\mathcal{B}$ induces the FOL formula:

$$\varphi_{\mathcal{B}} \coloneqq \exists \{v : v \in V(\varepsilon) \setminus U(\varepsilon)\} . \varphi_{E(\varepsilon)} \wedge \varphi_{H(\varepsilon)} \wedge \bigwedge_{n \in T \cap \mathbb{N}} \neg \varphi_{\mathcal{B}|_n}.$$

Example tree: 

Consider the byp (represented as a *flattened* byp, or *fyp*):



The FOL formula obtained is:

$$\neg \exists i, j . \neg \left(a(x, i) \wedge c(z, j) \right) \wedge \neg\neg \left(\neg b(i, y) \wedge \neg d(j, w) \right),$$

$$\equiv \forall i, j . \left(a(x, i) \wedge c(z, j) \right) \vee b(i, y) \vee d(j, w).$$

Satisfaction for branching hypergraphs

- An *assignment* from a byp $\mathcal{B}$ to a discrete hyp $\mathcal{H}$ is a function $\theta : \mathcal{B}(-1) \to V$.

Satisfaction for branching hypergraphs

- An *assignment* from a byp $\mathcal{B}$ to a discrete hyp $\mathcal{H}$ is a function $\theta : \mathcal{B}(-1) \to V$.
- The assignment *satisfies* the byp ($\theta \vDash \mathcal{B}$) if there is a morphism of hyps $\theta_\varepsilon : \mathcal{B}(\varepsilon) \to \mathcal{H}$ such that $\theta_\varepsilon \circ \iota = \theta$

Satisfaction for branching hypergraphs

- An *assignment* from a byp $\mathcal{B}$ to a discrete hyp $\mathcal{H}$ is a function $\theta : \mathcal{B}(-1) \to V$.
- The assignment *satisfies* the byp ($\theta \vDash \mathcal{B}$) if there is a morphism of hyps $\theta_\varepsilon : \mathcal{B}(\varepsilon) \to \mathcal{H}$ such that $\theta_\varepsilon \circ \iota = \theta$ and for all $n \in T \cap \mathbb{N}$, we have $\theta_\varepsilon \nvDash \mathcal{B}|_n$.

Satisfaction for branching hypergraphs

- An *assignment* from a byp $\mathcal{B}$ to a discrete hyp $\mathcal{H}$ is a function $\theta : \mathcal{B}(-1) \to V$.
- The assignment *satisfies* the byp ($\theta \vDash \mathcal{B}$) if there is a morphism of hyps $\theta_\varepsilon : \mathcal{B}(\varepsilon) \to \mathcal{H}$ such that $\theta_\varepsilon \circ \iota = \theta$ and for all $n \in T \cap \mathbb{N}$, we have $\theta_\varepsilon \nvDash \mathcal{B}|_n$.

Proposition

$\theta \vDash \mathcal{B}$ if and only if $\mathcal{H}$ is a model of $\varphi_{\mathcal{B}}$.

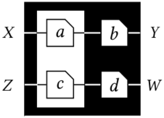
More is (tentatively) true

- Fyps give the 1-categorical part of the free *first-order bicategory*.

More is (tentatively) true

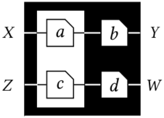
- Fyps give the 1-categorical part of the free *first-order bicategory*.
- The diagrammatic terms of (Bonchi, Di Giorgio, Haydon and Sobociński, 2024) are translated to fyps:

More is (tentatively) true

- Fyps give the 1-categorical part of the free *first-order bicategory*.
- The diagrammatic terms of (Bonchi, Di Giorgio, Haydon and Sobociński, 2024) are translated to fyps:we saw the translation of the term 

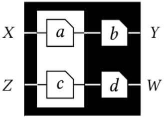
.

More is (tentatively) true

- Fyps give the 1-categorical part of the free *first-order bicategory*.
- The diagrammatic terms of (Bonchi, Di Giorgio, Haydon and Sobociński, 2024) are translated to fyps:we saw the translation of the term 

.
- What is missing is the 2-categorical part, i.e. the correct notion of a morphism between byps/fyps.

More is (tentatively) true

- Fyps give the 1-categorical part of the free *first-order bicategory*.
- The diagrammatic terms of (Bonchi, Di Giorgio, Haydon and Sobociński, 2024) are translated to fyps:we saw the translation of the term 

.
- What is missing is the 2-categorical part, i.e. the correct notion of a morphism between byps/fyps.
- We suspect we need to introduce some quotients in order to capture all morphisms.

References

- Arend Rensink. *Representing First-Order Logic Using Graphs*. ICGT 2004.
- Filippo Bonchi, Jens Seeber & Pawel Sobociński. *Graphical Conjunctive Queries*. CSL 2018.
- Filippo Bonchi, Alessandro Di Giorgio, Nathan Haydon & Pawel Sobociński. *Diagrammatic Algebra of First Order Logic*. LICS 2024.
- Arend Rensink & Andrea Corradini. *On Categories of Nested Conditions*. Springer LNCS 2024.

Thank you for your attention!