Modulation-Reaction Networks

Leo Lobski[1] and Yoàv Montacute[2]

[1]University College London, UK
[2]National Institute of Informatics, Japan

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Outline

Motivation: information flow vs. matter flow

- Reaction networks model the flow of *matter* or *entities*:

$$i_1 \xrightarrow{1} x \xrightarrow{0.7} o_1$$

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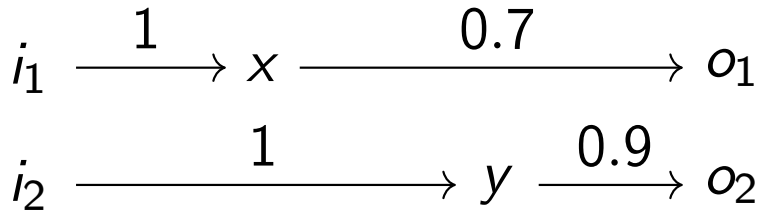

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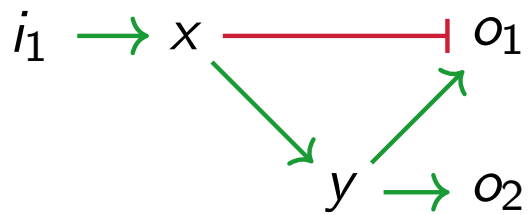

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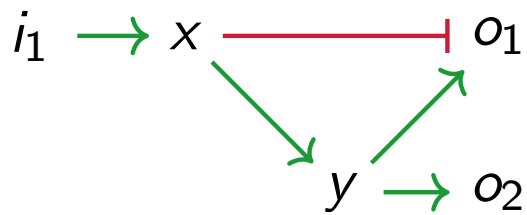
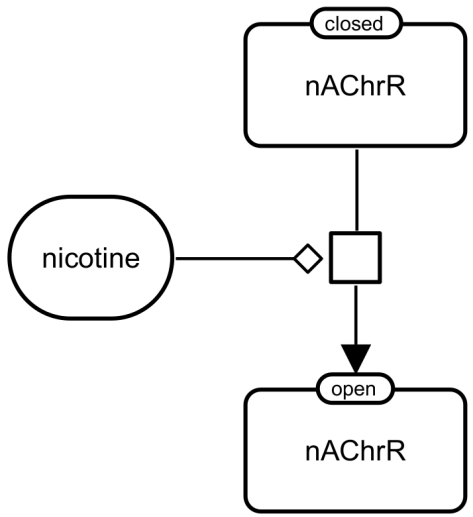
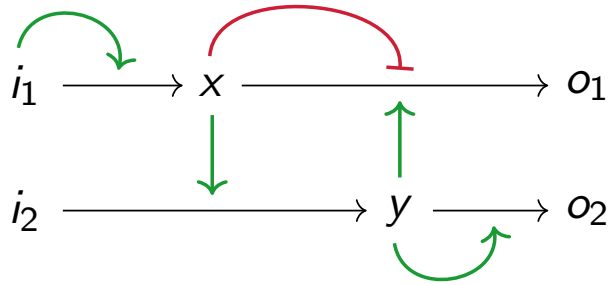


- Biochemical processes involve both:



MR-networks



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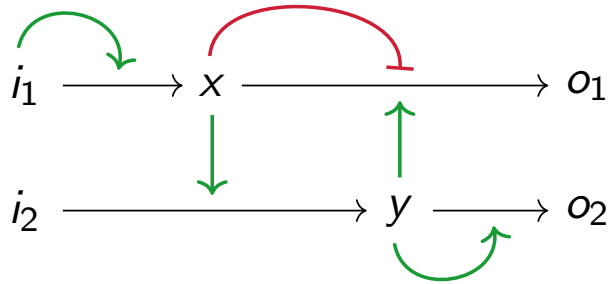

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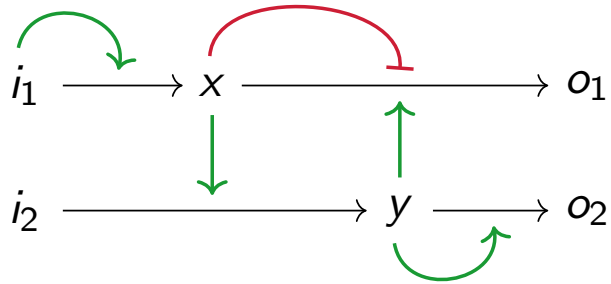

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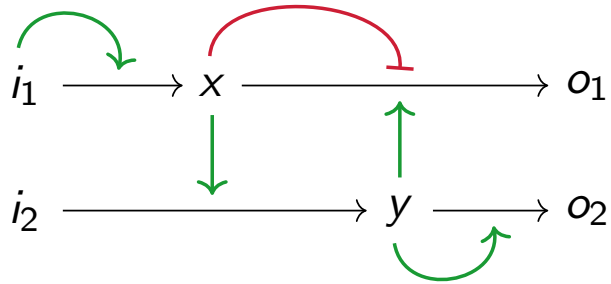

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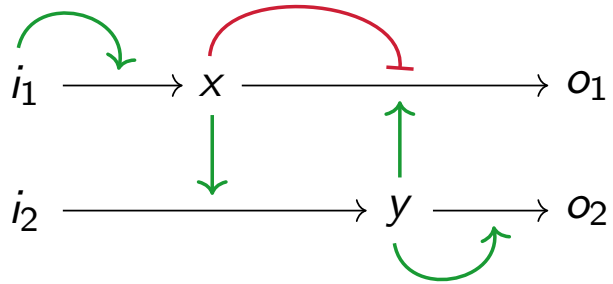

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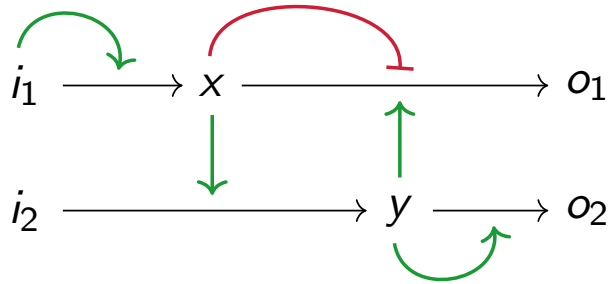

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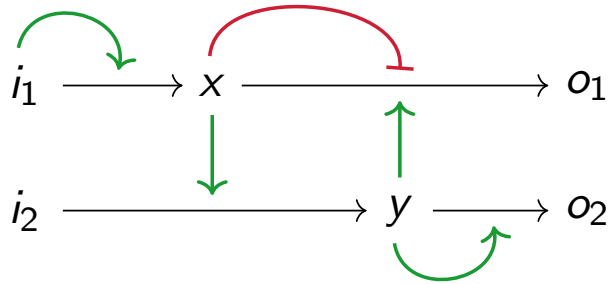

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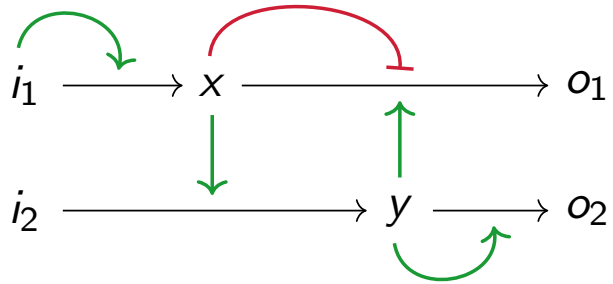

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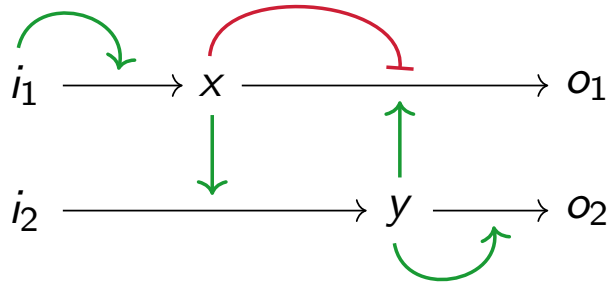

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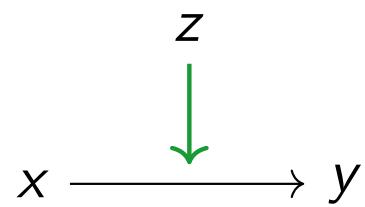

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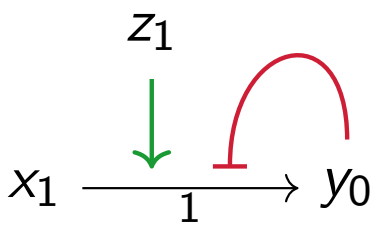

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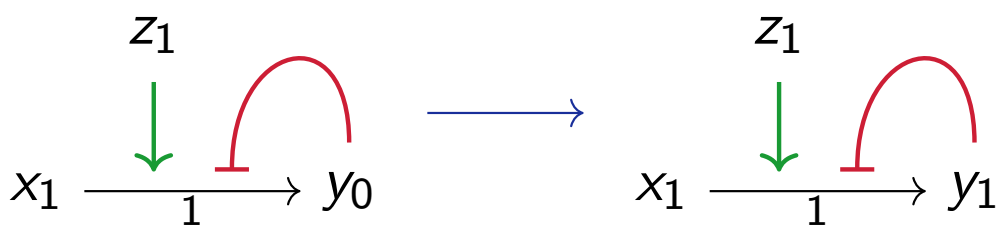

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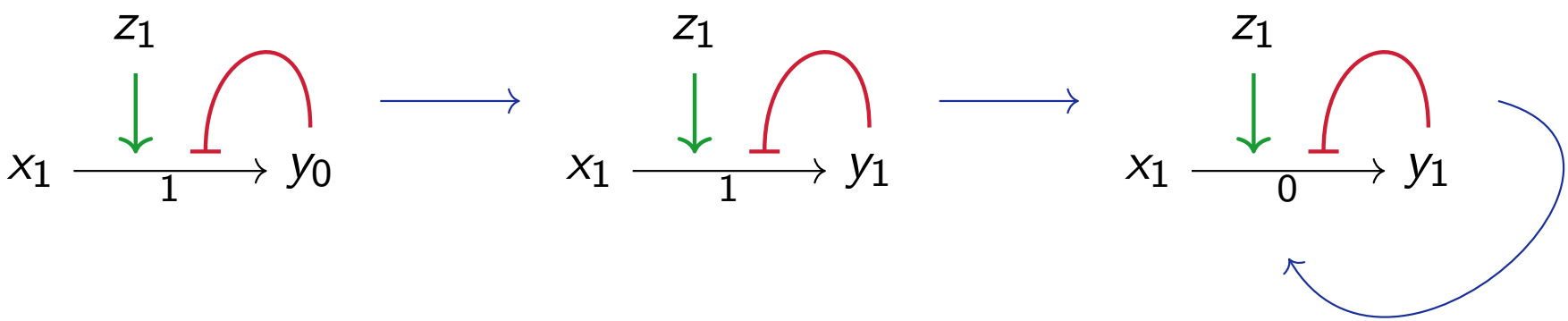

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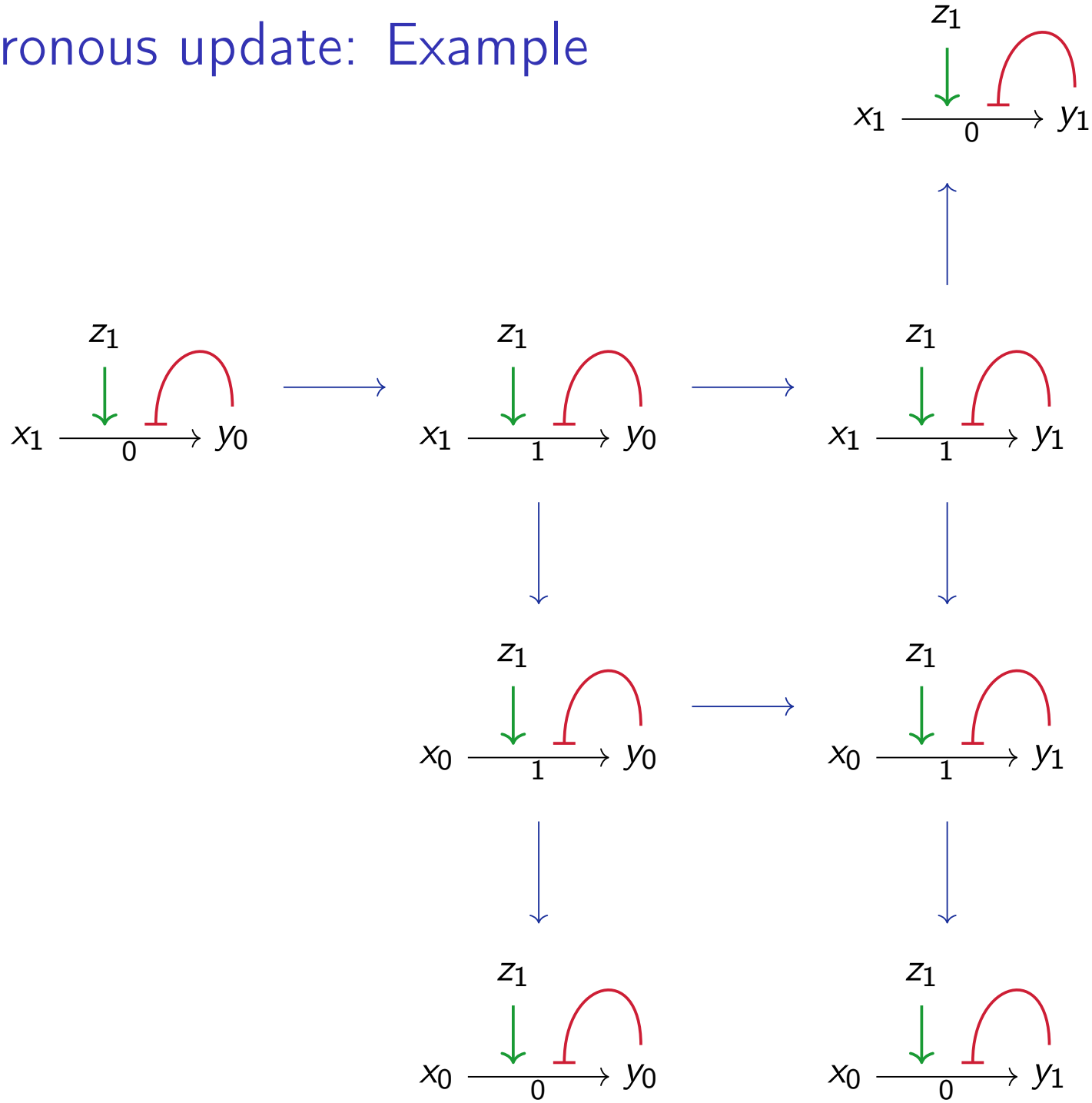

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- $@_i\varphi$ asserts that φ holds at the vertex named i

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Network modalities

$\mathcal{M}_i, x \vDash_\sigma j \iff f_i(x) = j, for j \in \mathbb{Z}_2$

$\mathcal{M}_i, x \vDash_\sigma p \iff (x, i) \in \sigma(p)$

$\mathcal{M}_i, x \vDash_\sigma \neg\varphi \iff \mathcal{M}_i, x \nvDash_\sigma \varphi$

$\mathcal{M}_i, x \vDash_\sigma \varphi \wedge \psi \iff \mathcal{M}_i, x \vDash_\sigma \varphi$ and $\mathcal{M}_i, x \vDash_\sigma \psi$

$\mathcal{M}_i, x \vDash_\sigma \Diamond^+\varphi \iff \exists y \in V$ s.t. $xRy, g_i(x, y) = 1$ and $\mathcal{M}_i, y \vDash_\sigma \varphi$

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Computation modalities

$\mathcal{M}_i, x \vDash_\sigma X\varphi \iff \mathcal{M}_{i+1}, x \vDash_\sigma \varphi$

$\mathcal{M}_i, x \vDash_\sigma F\varphi \iff \mathcal{M}_\ell, x \vDash_\sigma \varphi$ for some $\ell \geq i$

Fixed-point modality

$\mathcal{M}_i, x \vDash_\sigma \mu p.\varphi \iff (x, i) \in \bigcap \left\{ S \subseteq V \times \mathbb{N} : \Phi_\sigma^{\varphi,p}(S) \subseteq S \right\},$

where $\Phi_\sigma^{\varphi,p}(S) = \{(x', i') \in V \times \mathbb{N} : \mathcal{M}_{i'}, x' \vDash_{\sigma[p \mapsto S]} \varphi\}$

Hybrid modalities

$\mathcal{M}_i, x \vDash_\sigma y \iff x = y$

$\mathcal{M}_i, x \vDash_\sigma @_y\varphi \iff \mathcal{M}_i, y \vDash_\sigma \varphi$

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- $\mathsf{Reach}(x, y) := @_x\, \mu p.\, (y \vee F\Diamond^+ p)$

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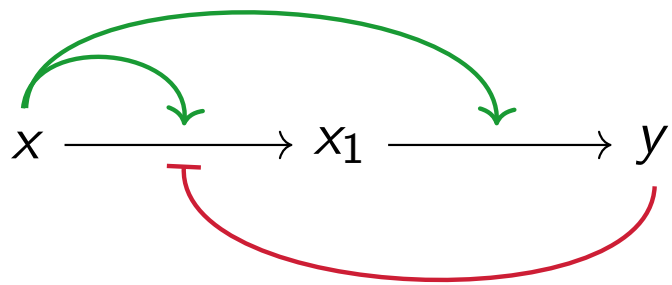
Example



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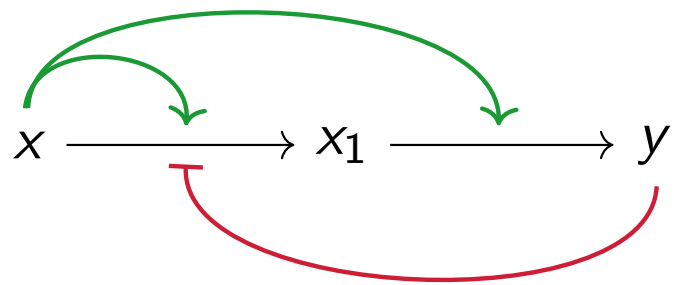

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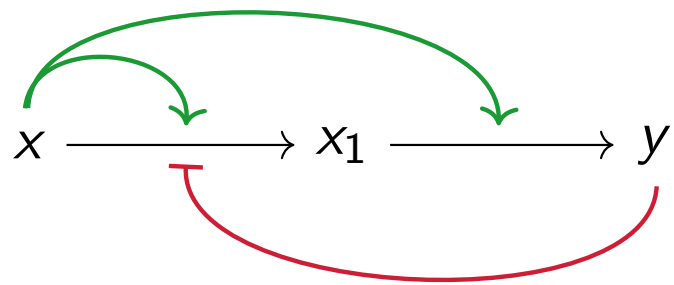

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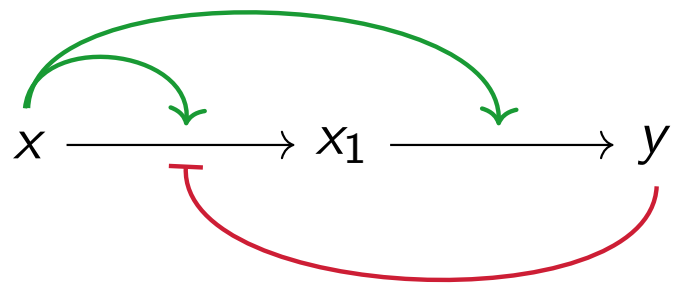

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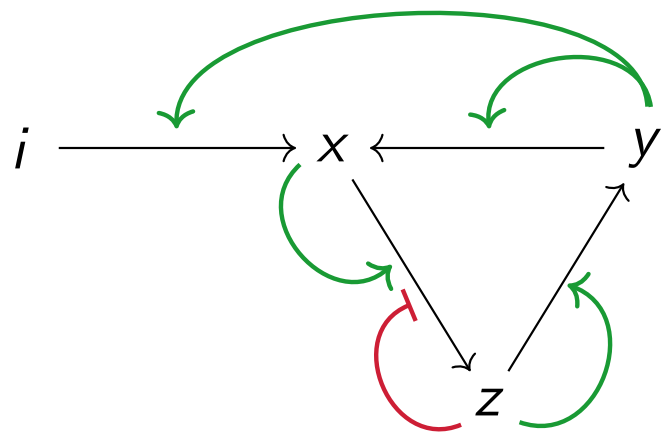

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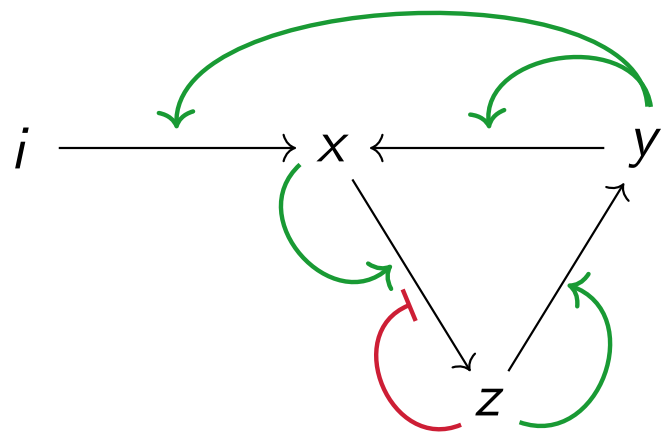

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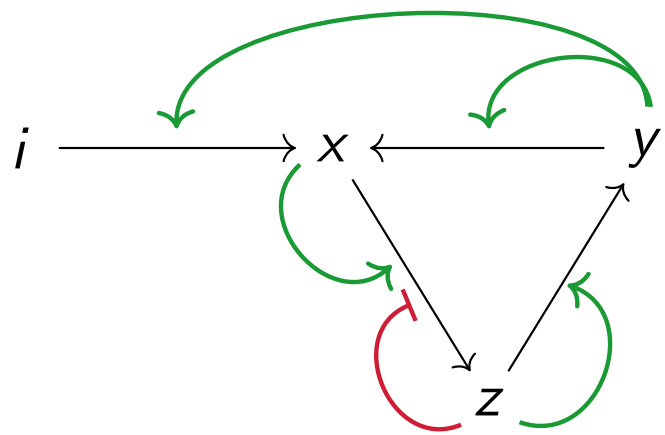

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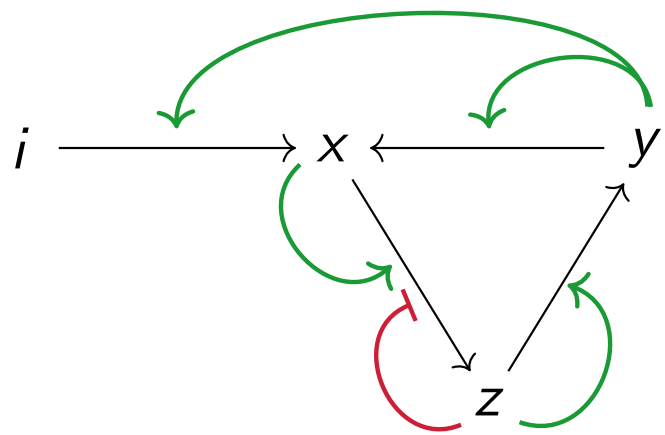

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Proposition

There is no MRL *sentence* φ *such that for all MR-models* $(\mathcal{M}, f_0)$ *one has* $\mathcal{M}_i, x \vDash \varphi \iff \left|\{y \in V : yRx, g_i(y, x) = 1\}\right| = 2$.

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Thank you for your attention!